Indian Statistical Institute, Bangalore
B. Math (Hons.) First Year
First Semester - Real Analysis I

Mid-Semester Exam | Date: 18th September 2023
Maximum marks: 30 | Duration: 2 hours

Answer any six and each question carries 5 marks

1. For $x, y \in \mathbb{R}$ with $x > 0$ prove that there exists $N \in \mathbb{N}$ such that $Nx > y$ and deduce that $1/n \to 0$.
2. For two sequences (x_n) and (y_n), prove that $\overline{\lim} x_n + \underline{\lim} y_n \leq \overline{\lim}(x_n + y_n)$.
3. Find $\underline{\lim}$ and $\overline{\lim}$ for the sequences: (a) $x_n = (-1)^{n+1} \frac{(n+2)}{(n+1)}$; (b) $y_n = n \sin(n\frac{\pi}{2})$.
4. If $a_n \to a$ and $b_n \to b$, prove that $a_n b_n \to ab$.
5. Prove that $a_n \to a \in \mathbb{R}$ if and only if $\overline{\lim} a_n = \underline{\lim} a_n \in \mathbb{R}$.
6. If (x_n) satisfies $|x_{n+1} - x_n| \leq \frac{1}{2}|x_n - x_{n-1}|$ for $n \geq 2$, prove that (x_n) converges.
7. Let (a_n) be a decreasing sequence of non-negative reals. Then the series $\sum a_n$ converges if and only if the series $\sum 2^k a_{2^k}$ converges.
8. If $\sum a_n$ converges, then $\sum(1 - 1/n)a_n$ and $\sum(2 + 1/n)a_n$ converge.